

A WORST-CASE APPROACH FOR ON-LINE FLUTTER PREDICTION

Rick C. Lind and Martin J. Brenner
 NASA Dryden Flight Research Center
 MS 4840 D/RS
 Edwards, CA 93523 USA
 email address: rick.lind@dfrc.nasa.gov
 email address: martin.brenner@dfrc.nasa.gov

19-03-TM
 065-21

Abstract. Worst-case flutter margins may be computed for a linear model with respect to a set of uncertainty operators using the structured singular value. This paper considers an on-line implementation to compute these robust margins in a flight test program. Uncertainty descriptions are updated at test points to account for unmodeled time-varying dynamics of the airplane by ensuring the robust model is not invalidated by measured flight data. Robust margins computed with respect to this uncertainty remain conservative to the changing dynamics throughout the flight. A simulation clearly demonstrates this method can improve the efficiency of flight testing by accurately predicting the flutter margin to improve safety while reducing the necessary flight time.

Key words: Aeroelasticity, Structural Dynamics, Flutter, Structured Singular Value, Robust Stability, Flight Test

1. Introduction

Flight flutter testing incurs dramatic time and costs associated with safely expanding the flight envelope to ensure no aeroelastic instabilities are encountered. NASA Dryden Flight Research Center is investigating methods to compute confident flutter margins to improve flight test efficiency [11].

Traditional pre-flight analytical methods of predicting flutter margins, such as the $p-k$ method, utilize a single model of the aircraft dynamics. These methods are not able to account for variations between the model and the aircraft so the predicted flutter margins may be overly optimistic.

Traditional flight test methods of determining flutter margins often utilize tracking estimates of modal damping obtained from flight data [6]. These methods are both inefficient and dangerous. Multiple data sets must be taken at each test point to ensure the critical modal dynamics have been sufficiently excited. Also, the envelope must be expanded in small increments since damping only indicates stability at the current test point and cannot be extrapolated to consider other flight conditions.

Several proposed on-line methods use modal filtering, parameter identification and envelope analysis to increase confidence in the flutter boundary [3]. These methods, like damping tracking, compute stability indicators rather than predictors.

A novel method for computing flutter margins is developed based on robust stability theory [8]. Uncertainty operators are associated with a linear model to account for modeling errors. The structured singular value, μ , computes a stability margin robust to these uncertainties. In this respect, a worst-case flutter margin is computed.

This paper considers an on-line implementation using μ , a stability predictor, to replace damping. This method presents advantages over traditional flutter analysis methods since it combines a well developed analytical model with flight data describing the true aircraft, as shown in Figure 1.

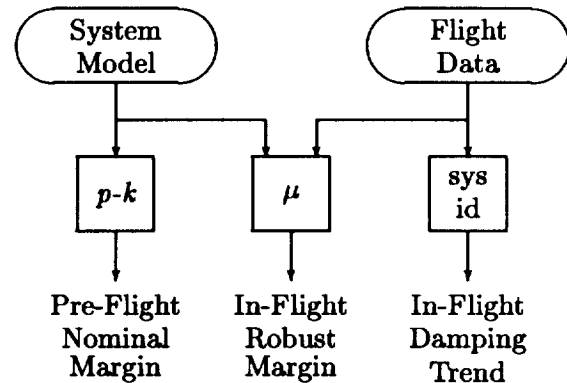


Figure 1. Methods for Computing Flutter Margins

A simulated flight test demonstrates several significant advantages of the μ method as compared to traditional methods. The μ method is able to generate realistic flutter margins using flight data from a stable flight condition; whereas little information is obtained from tracking damping until the onset of flutter. Also, the robust flutter margins are consistently conservative due to model validation of the uncertainty set using flight data; whereas nominal margins computed with $p-k$ do not account for changes in aircraft dynamics.

The concept of a "flutterometer" is introduced as a flight test tool to indicate the proximity of a flutter condition. The simulation demonstrates a feasible concept of such a tool based on robust flutter margin predictions which remain conservative to time-varying aircraft dynamics.

2. Robustness Measure μ

Any aeroelastic model is an approximate representation of the aircraft dynamics. Uncertainty operators are associated with the model in the framework of the structured singular value, μ , to account for inaccuracies such as errors and unmodeled dynamics.

The uncertainty operator, Δ , is allowed to lie within a norm bounded set. Weighting matrices are usually included in the μ framework to normalize the uncertainty norm bound to unity.

$$\Delta = \{\Delta : \|\Delta\|_\infty \leq 1\}$$

Define μ with respect to the set Δ .

$$\mu(P) = \frac{1}{\min \{\bar{\sigma}(\Delta) : \Delta \in \Delta, \det(I - P\Delta) = 0\}}$$

with $\mu(P) = 0$ if there exists no $\Delta \in \Delta$ such that $\det(I - P\Delta) = 0$.

Define elements of P as transfer function from disturbances to errors P_{22} , transfer function relating uncertainty feedback P_{11} , and similar P_{12} and P_{21} .

μ is an exact measure of robustness for systems with structured uncertainty. The inverse of μ can be interpreted as a measure of the smallest destabilizing perturbation. The system is guaranteed to be robustly stable for all uncertainty operators bounded by the smallest destabilizing value [4].

Theorem 2.1. A stable system P is robustly stable for all $\Delta \in \Delta$ if and only if $\mu(P_{11}) < 1$.

Model validation algorithms can verify the uncertainty description is sufficient to describe variations between the model and measured data. A μ -analysis method of model validation has been developed that determines if there exists some perturbation to the system dynamics, within the allowed set of uncertainty, for which the robust system could produce the measurement data for the indicated input data [7].

Theorem 2.2. Given measurements y generated by input u , then a system P with uncertainty Δ is not invalidated if $\mu(P_{11} - P_{12}u(P_{22}u - y)^{-1}P_{21}) > 1$

Unfortunately, μ is difficult to compute. Upper and lower bounds have been derived which may be solved efficiently using convex optimization and a power iteration [1].

3. Robust Flutter Model

Consider the aeroelastic equation of motion [5].

$$M\ddot{\eta} + C\dot{\eta} + K\eta + \bar{q}Q(s)\eta = 0 \quad (1)$$

where the unsteady aerodynamic forces are

$$Q(s) = D_Q + C_Q(sI - A_Q)^{-1}B_Q \quad (2)$$

This system is parameterized around \bar{q} such that the μ value describes the smallest variation in dynamic pressure for the onset of flutter. This is accomplished by considering an additive perturbation, $\delta_{\bar{q}}$, on the nominal dynamic pressure \bar{q}_o .

$$\bar{q} = \bar{q}_o + \delta_{\bar{q}} \quad (3)$$

The nominal plant P_{nom} , based on Equations 1 and 2, incorporates the perturbation $\delta_{\bar{q}}$ as a linear fractional transformation using signals z and w where $w = \delta_{\bar{q}}z$ is the perturbation feedback relationship. Stability with respect to a range of $\delta_{\bar{q}}$ values is analyzed with the robust stability condition of Theorem 2.1 with $P_{nom}^{(11)}$, such that $z = P_{nom}^{(11)}w$, is given below. Define $\hat{M} = -M^{-1}$.

$$P_{nom}^{(11)} = \left[\begin{array}{ccc|c} 0 & I & 0 & 0 \\ \hat{M}(K + \bar{q}_o D_Q) & \hat{M}C & \bar{q}_o \hat{M}C_Q & -I \\ B_Q & 0 & A_Q & 0 \\ -\hat{M}D_Q & 0 & -\hat{M}C_Q & 0 \end{array} \right] \quad (4)$$

The full nominal plant, P_{nom} , utilizes $P_{nom}^{(11)}$ with additional elements to describe the generation of measurements and effects of force excitation.

A robust model P_{rob} associates uncertainty operators with P_{nom} to account for modeling errors between the theoretical system and the physical aircraft. These operators may enter as parametric uncertainties to reflect variations in specific elements such as mass and damping or dynamic uncertainties to allow unstructured variations in magnitude and phase of signals.

Aeroelastic flight data, if available, can be used to determine a realistic uncertainty description for P_{rob} using Theorem 2.2. Utilizing flight data may increase confidence in the robust model; however, it may also introduce additional uncertainty due to flight test issues, aircraft performance, aeroelastic excitation and frequency domain mapping [11, 9].

Robust flutter margins are computed for P_{rob} using the robust stability condition of Theorem 2.1. A robust flutter margin is the smallest destabilizing perturbation, $\delta_{\bar{q}}$, to dynamic pressure for P_{rob} with the given amount of uncertainty. This is the worst-case margin for the range of considered dynamics.

4. Robust Flutter Parameter

The flutter computation method described in this paper uses μ as the worst-case flutter parameter. There are several advantages to using μ as the flutter parameter. μ is a much more informative flutter margin as compared to traditional parameters such as pole location and modal damping.

The conservatism introduced by considering the worst-case uncertainty perturbation can be interpreted as a measure of sensitivity. Robust μ values which are significantly different than the nominal flutter margins indicate the plant is highly sensitive to modeling errors and changes in flight condition. A small perturbation to the system can drastically alter the flutter stability properties. Conversely, similarity between the robust and nominal flutter margins indicates the aircraft is not highly sensitive to small perturbations.

Robustness analysis determines not only the norm of the smallest destabilizing perturbation but also the direction. This information relates exact perturbations for which the system is particularly sensitive. μ can thus indicate the worst-case flutter mechanism which may naturally extend to indicate active and passive control strategies for flutter suppression.

Additionally, damping is only truly informative at the point of instability since stable damping and damping trends at a given flight condition do not guarantee what increases in dynamic pressure may be safely considered. μ computes the smallest destabilizing perturbation which indicates the nearest flight conditions that will cause a flutter instability. In this respect, μ is a stability predictor while damping is merely a stability indicator.

These characteristics of μ make the worst-case flutter algorithm especially valuable for flight test programs. Aeroelastic flight data can be measured at a stable flight condition and used to evaluate uncertainty operators. The μ method, unlike damping estimation, does not require the aircraft to approach instability for accurate prediction. μ can be computed to update the stability margins with respect to the new uncertainty levels. The worst-case stability margin then indicates what flight conditions may be safely considered for safe and efficient expansion of the flight envelope.

5. On-Line Implementation

Traditional flight flutter testing for envelope clearance proceeds with several repeated operations [6]. The aircraft is trimmed at a stable flight condition on the edge of the envelope where measured aeroelastic responses are analyzed to estimate damping and modal characteristics. If the estimates do not indicate the imminent onset of flutter, the envelope is slightly expanded and the procedure repeats until flutter or a performance limit is reached.

Worst-case flutter margins can easily be computed on-line in such a flight test program. The implementation can compute these margins in parallel with the damping estimates. Both methods can utilize the same data to update the stability measure at each test point. The robust model, which uses the same nominal plant throughout the flight, simply updates the uncertainty set such that the model is not invalidated by the flight data and then computes the worst-case margin with respect to that uncertainty.

Several robust flutter margins may be computed at a test point by computing margins which are worst-case to different uncertainty sets. Two obvious approaches are denoted here as local or global.

A local approach computes an uncertainty set at each test point such that the model is not invalidated by the current flight data. No knowledge of past uncertainties or data is used. This approach may seem logical considering a linear transonic model, which is often suspect, associates an uncertainty description which may be overly conservative for a subsonic model. This method may be highly accurate but could be overly optimistic if critical dynamics are not sufficiently excited.

A global approach computes flutter margins with respect to the largest uncertainty required for any point in the flight. It may be reasonable that if a model has a certain level of error at a given flight condition, then that same level might exist at all flight conditions. This method may be more conservative but the safety of the margin is increased since it is less susceptible to poor excitation at a certain test point.

Many flight analysis facilities are increasing computational resources to decrease required flight time. Such environments make on-line analysis of robust flutter margins extremely attractive. Separate computers can be used so worst-case flutter margins do not replace damping estimates; rather, they augment traditional flutter measures.

6. Simulation : Aircraft Model

A supersonic flight flutter test is simulated to demonstrate the benefits of utilizing the robust flutter margin algorithm in an on-line manner.

The F/A-18 Systems Research Aircraft (SRA) provides the aircraft model for the simulated flight test. This aircraft is a two-seat configuration fighter with production engines being flown at NASA Dryden Flight Research Center [2].

There are three linear state-space models used in the simulation.

- P_{nom} : nominal theoretical model
- P_{rob} : robust theoretical model
- P_{true} : true aircraft dynamics

The nominal theoretical model, P_{nom} , is a structural model coupled with unsteady aerodynamic forces. P_{nom} is parameterized around dynamic pressure using an operator $\delta_{\bar{q}}$ as in Equation 4. This model was used for flutter analysis of the F/A-18 SRA using the μ method and traditional algorithms such as the p - k method [8, 10, 12].

P_{nom} is a 84 state plant for this simulation to model the 14 symmetric structural modes of the aircraft along with the 56 states used in the state-space representation of the unsteady aerodynamic forces.

The robust theoretical model, P_{rob} , associates an uncertainty description with the nominal plant to describe errors and unmodeled dynamics. Two uncertainty operators, Δ_A and Δ_{in} , are included in the robust model to account for variations between the nominal plant and the true aircraft dynamics.

The uncertainty operator Δ_A affects the state matrix of the nominal plant to model variations in both natural frequency and damping for each mode. Δ_A is a structured diagonal matrix with real scalar parameters as elements. Separate scalars are used to affect each modal response and time lag in the state matrix. A scalar associated with a modal response is repeated two times while each time lag uncertainty appears once on the diagonal.

Initial weights for these uncertainties are chosen based on previous analysis of P_{nom} at supersonic flight conditions [8, 10]. The weighting W_ω describes the percent of variation allowed in natural frequency while W_ζ relates to damping and W_l is associated with time lags in the unsteady aerodynamic forces.

$$W_\omega = .05 \quad W_\zeta = .15 \quad W_l = .15 \quad (5)$$

The uncertainty operator Δ_{in} is a complex multiplicative uncertainty on the excitation force. A weighting function, W_{in} , reflects the frequency varying levels of multiplicative uncertainty such as a large component at high frequency to indicate no dynamics above 40 Hz are included in the model.

$$W_{in} = 5 \frac{s + 100}{s + 5000} \quad (6)$$

P_{rob} also contains sensor noise with a magnitude up to 10% of the measurement signal.

The block diagram for robust flutter margin analysis of the F/A-18 SRA model P_{rob} , including the parametric variation in dynamic pressure $\delta_{\bar{q}}$ and uncertainties Δ_A and Δ_{in} , is given in Figure 2.

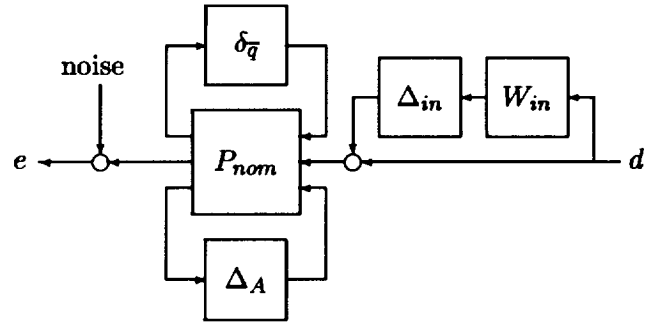


Figure 2. Robust Model, P_{rob} , of the F/A-18 SRA

The model P_{true} is used as the ‘true’ F/A-18 SRA aeroelastic dynamics. P_{true} is based on the nominal theoretical plant; however, P_{nom} has two major inaccuracies due to changes in the dynamics of the true aircraft. The type and magnitude of the errors in P_{nom} are chosen from within a set of modeling errors determined from post-flight analysis of recent flight tests with the F/A-18 SRA [10].

- error in structural damping
- unmodeled time-varying mass

The structural damping of P_{true} is 10% less than that of P_{nom} . This is introduced by a constant scaling of 0.9 to the C matrix term of Equation 1.

The mass of the true aircraft is time-varying throughout the simulation while the nominal plant is formulated using a constant heavyweight condition. The dynamics of P_{true} are more realistic than P_{nom} since mass may vary significantly during actual flight testing.

The Trailing Edge Flap mode for P_{true} at Mach 1.2 is the critical flutter mode. Flutter conditions vary from 27.36 Hz for $\bar{q} = 2360 \text{ lb/ft}^2$ with P_{true} at 95% heavyweight mass to 27.38 Hz for $\bar{q} = 2255 \text{ lb/ft}^2$ at 90% heavyweight mass.

7. Simulation : Flight Test

The simulation is designed to model the procedures used in a real flight flutter test. For ease of presentation, only the simulated flight test to determine the flutter boundary at Mach 1.2 for the symmetric modes is discussed. Flutter clearance of the entire flight envelope including antisymmetric modes is a straightforward extension to this simulation.

The flight test procedure involves iterating over several steps.

1. decrease altitude to raise \bar{q} by 100 lb/ft^2
2. measure symmetric response data
3. determine an uncertainty set such that the flight data does not invalidate the model
4. compute robust flutter margin

The initial flight condition is $\bar{q} = 100 \text{ lb/ft}^2$ at the altitude of 68.8 kft . The aircraft proceeds to the first test point at $\bar{q} = 700 \text{ lb/ft}^2$ which corresponds to an altitude of 27.7 kft . Further test points occur at intervals of 100 lb/ft^2 .

The aircraft follows the same iterative procedure throughout the simulation until the onset of a flutter instability. Naturally during a real flight test the aircraft would not purposely approach a flutter margin so closely, but encountering the flutter condition serves to demonstrate the accuracy of the computed flutter margins for the simulation.

The total flight path is given in Figure 3.

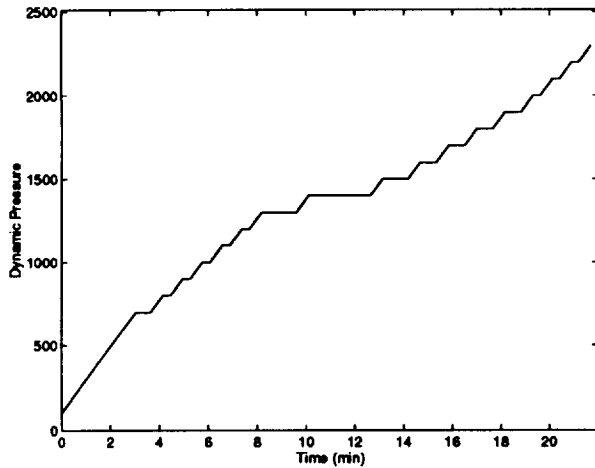


Figure 3. Flight Path at Constant Mach=1.2

Time is not a commanded variable in this simulation so the length of the flight test is determined by the computational analysis time. The only known function of time is the flight path between test points. The aircraft is instructed to increase the dynamic pressure by 100 lb/ft^2 and stabilize at Mach 1.2 in 30 seconds.

The horizontal portions of the flight path indicate time spent at a test point for which the flight conditions do not change. The length of time at each test point is determined by the computational cost of modal validation and robust flutter margin analysis. The unequal computational times at different test points demonstrated in Figure 3 is due to variations in the number of iterations required to generate and validate increased uncertainty levels.

The true aircraft dynamics are time-varying due to decreasing mass throughout the simulation. The nominal and robust plant models used for flutter margin prediction are formulated for the heavy-weight flight condition and thus are progressively worse representations of the true aircraft dynamics. Consequently, the amount of uncertainty associated with the robust model must increase to ensure the allowed range of dynamic variations includes the true dynamics.

The mass of the aircraft P_{true} is a time-varying element of the simulation. The initial mass is chosen as 95% of the heavyweight condition of the SRA with full fuel. The mass variation proceeds throughout the simulation by decreasing at a rate of 5% of the heavyweight value for 20 minutes of flight time. Figure 4 shows the mass of P_{true} throughout the simulation.

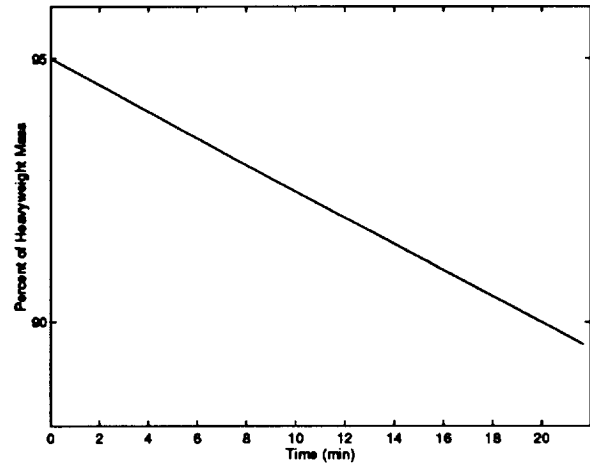


Figure 4. Mass of the Aircraft P_{true} Expressed as Percentage of Heavyweight Condition

The time-varying mass will affect every aeroelastic modal response. The decrease in mass will appear as increases in natural frequency and damping. The modal characteristics for the Trailing Edge Flap mode of P_{true} at $\bar{q} = 1000 \text{ lb/ft}^2$ are $\omega = 25.68 \text{ Hz}$ and $\zeta = .0125$ for 95% of heavyweight mass while $\omega = 25.74 \text{ Hz}$ and $\zeta = .0131$ for 90% of heavyweight mass. P_{rob} must be robust to these changes in modal dynamics and the corresponding change in flutter margins of P_{true} .

The uncertainty levels of P_{rob} are analyzed at each test point to ensure the measured flight data does not invalidate the model. If the robust model is invalidated, the uncertainty weights are iteratively increased and analyzed until the amount of uncertainty is sufficient to model the observed variations between P_{true} and P_{rob} .

This simulation increases the amount of modal uncertainty to ensure the flight data could be generated by the robust model while keeping the amount of input multiplicative uncertainty constant. The largest increases in uncertainty are for the damping levels with smaller increases associated with natural frequency uncertainty. The amount of uncertainty in damping is expressed as a percent of the nominal damping value in Figure 5.

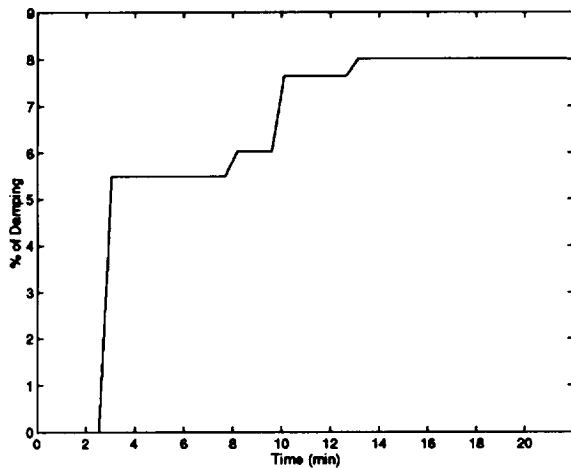


Figure 5. Level of Modal Damping Uncertainty

The initial uncertainty levels in the simulation are pre-defined based on previous analysis of the F/A-18 SRA aircraft and models. Figure 5 demonstrates this initial uncertainty level of approximately 5% is sufficient to validate the robust model according to flight data recorded between 3 and 8 minutes of the simulation.

Increases in the damping uncertainty are generated at several points between the 8th and 14th minutes of the simulation. The amount of these increases is determined solely by the model validation algorithm. A final level of 8% is required to validate the model at the flight condition of 18 minutes. This level agrees with Figure 8 which shows a variation in damping between the nominal and true plants of approximately 8% at this flight condition.

The modal validation portion of the flutter analysis was stopped due to low damping values of less than .005 after 19 minutes. The algorithm encounters some conditioning problems when the system is extremely close to an instability.

8. Simulation : On-Line Flutter Margins

Flutter margins are computed at each test point based on the levels of uncertainty required to validate the model with flight data from that test point. Figure 6 demonstrates these calculated robust and nominal flutter margins along with the true flutter margin.

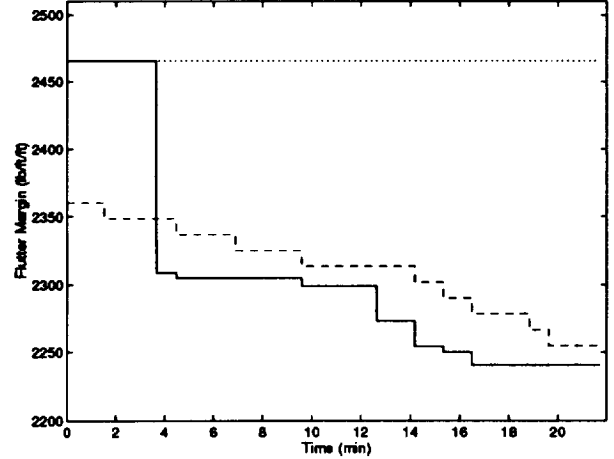


Figure 6. Flutter Margins expressed as Dynamic Pressure Flight Condition : Robust μ Prediction (—), Nominal μ Prediction (.), True Flutter Margin (---)

The robust flutter margin is not computed with respect to any uncertainty for the first 3 minutes so the nominal and robust margins are identically poor at first. The nominal margin does not account for any uncertainty and remains constant throughout the flight. The robust margin varies with time due to changes in the uncertainty levels required to validate the flight data at each test point.

The robust margin decreases to remain conservative to the true flutter margin due to the worst-case nature of the computation with respect to the increasing uncertainty levels. Note it is only conservative to the flutter margin for P_{true} at that particular test point since the uncertainty set at time t_1 does not necessarily cover the dynamics at a later time t_2 due to the time-varying mass variation.

The μ -analysis method of computing robust flutter margins is able to utilize flight data from a stable test point. The robust flutter margin immediately provides a realistic flutter margin without requiring the aircraft to approach an unstable flight condition.

The nominal flutter margin is shown on Figure 6 to demonstrate the large error in this flutter margin as compared to the final true flutter condition. This nominal margin corresponds to the pre-flight prediction computed via μ or $p-k$ methods with no accounting for modeling uncertainty.

The concept of a flutterometer is introduced as a tool for indicating the proximity to a flutter condition. One implementation of this concept is to describe the smallest increase in \bar{q} which results in flutter. Figure 7 demonstrates this flutterometer concept for the simulated flight test.

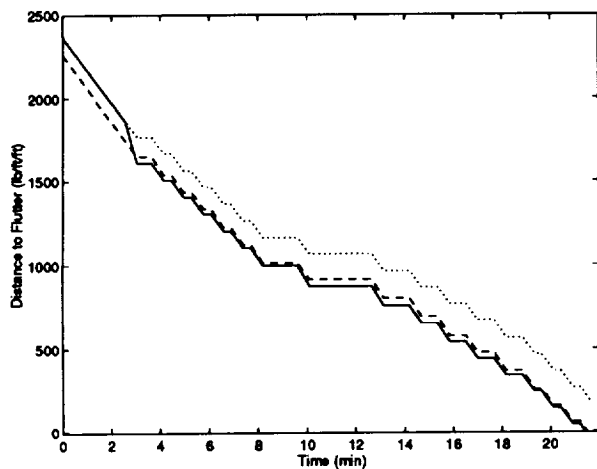


Figure 7. Flutterometer showing Distance to Flutter in Dynamic Pressure : Robust μ Prediction (—), Nominal μ Prediction (···), True Distance to Flutter (---)

The benefits of the μ based flutterometer are easily seen when considering the traditional method of predicting flutter margins based on the evolution of modal damping as shown in Figure 8.

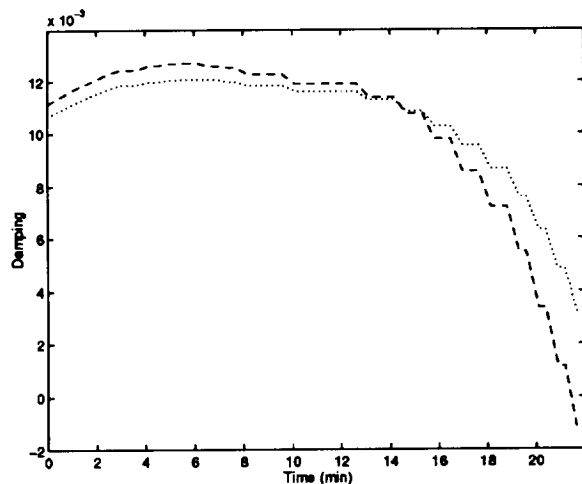


Figure 8. Modal Damping for the Trailing Edge Flap Mode : Nominal Model P_{nom} (···), True Aircraft (---)

Tracking modal damping is clearly inferior to the μ method for on-line flutter prediction. Modal damping provides little information until minute 17 when the aircraft is approaching flutter. After that time, damping shows a decreasing trend to indicate proximity to an instability; however, the nonlinear behavior precludes calculating the exact proximity of that instability.

9. Simulation : Computational Cost

The efficiency of any flight flutter test program is directly measured by the ability to compute accurate flutter margins in a minimal amount of flight time without sacrificing safety of the pilot and aircraft. This simulation demonstrates computing robust flutter margins can be efficiently performed in an on-line manner for flight test programs.

The robust flutter margin procedure is implemented in Matlab using the μ -Analysis and Synthesis Toolbox [1]. The computation times presented are for a standard 200 MHz Pentium computer often available in a flight analysis facility.

The computation time at each test point results from validating the uncertainty description and computing robust flutter margins. These computation times are shown in Figure 9 with circles representing the analysis time at a specific test point.

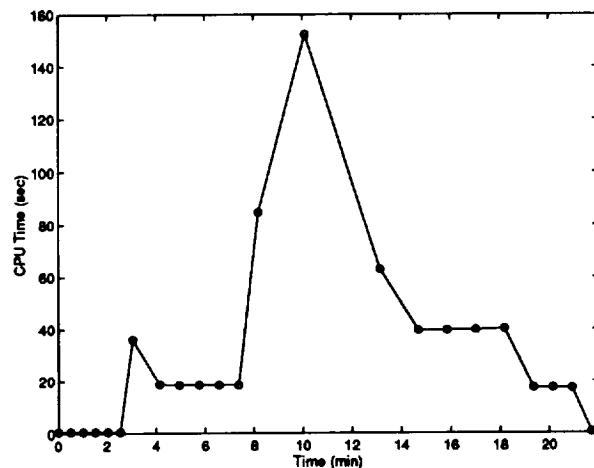


Figure 9. Computational CPU Time to Determine Robust Flutter Margin at each Test Point (o)

The 3rd minute test point marks the first robust flutter margin computation. This initial uncertainty set is conservative to the time-varying dynamics until the 8th minute, as evidenced by the constant computational cost. Similarly, the uncertainty set at the 14th minute is conservative until the 18th minute. The computational cost is reduced after this time because model validation is no longer performed due to the onset of flutter instability.

The 8,10 and 13 minute test points require longer time than other test points. Figure 5 shows the uncertainty at these three test points is increased to account for the time-varying dynamics of P_{true} . The additional analysis time corresponds to the extra iterations required for invalidating the old uncertainty levels and computing new levels which are not invalidated by the flight data.

The flight path shown in Figure 3 combined with Figure 9 is indicative of the efficiency of the robust algorithm; however, there are several aspects to a real flight test program that are not easily modeled. Firstly, the simulation did not take into account data transfer time between the aircraft and the analysis engines [6]. Also, the simulation assumed a single data set generated at each test point could be used for model validation while typically several sets are required to generate a rich flight data set [9]. These issues are demonstrative of inefficiencies associated with flight data which are being addressed through improved flight test procedures and distributed analysis environments [11].

Given a good data set, the robust flutter margins are computed for the majority of test points in less than 1 minute with the largest analysis time being only 2.5 minutes. This clearly does not present an excessive computational burden to the flight program. In fact, the benefits of the procedure are more emphasized when considering the analysis time to generate this accurate predictor is comparable to the computational time required for traditional damping estimates which provide far less information about the true flutter boundary.

10. Conclusion

Worst-case flutter margins computed with a robust stability method can provide valuable information about the flutter dynamics of an aircraft. This paper introduces an on-line implementation of such a method for use in a flight flutter test program.

A simulated flight test demonstrates several advantages to the robust flutter margin method as compared to traditional flutter analysis methods. The robust method uses flight data from a stable flight condition to compute a flutter boundary without requiring the aircraft to approach the instability. Also, this method computes margins which remain conservative to time-varying dynamics by updating uncertainty levels at every test point so the new flight data does not invalidate the model.

The robust flutter margins quantify the proximity to a flutter condition which can not be computed by tracking damping trends. This fact leads to the first formulation of a robust flutterometer tool. This method could dramatically increase efficiency of a flight test program by accurately predicting flutter margins despite unmodeled time-varying dynamics of the aircraft while increasing safety due to the conservativeness of the predictions.

11. Acknowledgments

The authors wish to acknowledge the financial support of the Structures Branch of NASA at Dryden Flight Research Center. Particular appreciation is extended to Larry Freudinger for his ideas towards on-line flutter estimation, Dave Voracek for his knowledge of the F/A-18 SRA, and Leonard Voelker for his understanding of flutter. Dr. Lind is supported through the PostDoctoral Fellowship program of the National Research Council.

References

1. Balas, G., Doyle, J., Glover, K., Packard, A. and Smith, R., *μ -Analysis and Synthesis Toolbox - Users Guide*, The MathWorks, Natick, MA, 1991.
2. Brenner, M., Lind, R. and Voracek, D., "Overview of Recent Flight Flutter Testing Research at NASA Dryden," *1997 AIAA Structures, Structural Dynamics and Materials Conference*, Orlando, FL, AIAA-97-1023, April 1997.
3. Cooper, J., Desforges, M., Emmett, P. and Wright, J., "Advances in the Analysis of Flight Flutter Test Data," *Proceedings of the 80th AGARD Structures and Materials Panel*, AGARD-CP-566, Rotterdam, The Netherlands, May 8-10 1995, pp. 13:1-13:12.
4. Doyle, J., "Analysis of Feedback Systems with Structured Uncertainty," *IEEE Proceedings*, Part D, vol. 129, November 1982, pp. 242-250.
5. Gupta, K., Brenner, M. and Voelker, L., *Development of an Integrated Aeroservoelastic Analysis Program and Correlation with Test Data*, NASA TP-3120, May 1991.
6. Kehoe, M., "A Historical Overview of Flight Flutter Testing," *Proceedings of the 80th AGARD Structures and Materials Panel*, AGARD-CP-566, Rotterdam, The Netherlands, May 8-10 1995, pp. 1:1-1:15.
7. Kumar, A. and Balas, G., "An Approach to Model Validation in the μ Framework," *Proceedings of the 1994 American Controls Conference*, Baltimore MD, June 1994, pp. 3021-3026.
8. Lind, R. and Brenner, M., "Robust Flutter Margins of an F/A-18 Aircraft from Aeroelastic Flight Data," *AIAA Journal of Guidance, Control and Dynamics*, scheduled to appear July/August 1997.
9. Lind, R. and Brenner, M., "Incorporating Flight Data into a Robust Aeroelastic Model," in review for *AIAA Journal of Aircraft*, submitted October 1996.
10. Lind, R. and Brenner, M., "Worst-Case Flutter Margins from F/A-18 Aircraft Aeroelastic Data," *1997 AIAA Structures, Structural Dynamics and Materials Conference*, Orlando, FL, AIAA-97-1266, April 1997.
11. Lind, R., Brenner, M. and Freudinger, L., "Improved Flight Test Procedures for Flutter Clearance," *Proceedings of the 1997 CEAS International Forum on Aeroelasticity and Structural Dynamics*, Rome, Italy, June 1997.
12. Voelker, L., *F-18/SRA Flutter Analysis Results*, NASA TM, in preparation, 1997.